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1992 J. Phys. A: Math. Gen. 25 L529

(http://iopscience.iop.org/0305-4470/25/9/005)

J. Phys. A: Math. Gen. 25 (1992) L529–L533. Printed in the UK

LETTER TO THE EDITOR

The time evolution operator of an atom in a deformed intense field

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Received 14 January 1991

Abstract. We explicitly calculate the time evolution operator of an atom in a deformed electromagnetic field which is dependent on deformation parameter and possesses quantum group symmetry. The density operator and the time variation of photon distribution are also given, both with and without intensity-dependent coupling.

With the recent successful observations of squeezed states of the electromagnetic field, a great deal of attention has been given to the investigations of radiation–matter interactions [1-5]. Chaichain, Ellinas and Kulish [4] introduced the effects of q-deformation in the Jaynes–Cummings model (JCM) [6]. Celeghini, Rasetti and Vitiello [5] discussed the squeezing and quantum algebra and showed that the q-deformed harmonic oscillator generates squeezing when the Hermitian operators P_q and Q_q are introduced as

$$P_q = \mathrm{i}\left(\frac{m\hbar\omega}{2}\right)^{1/2}(b_q - b_q^+)$$
$$Q_q = \left(\frac{\hbar}{2m\omega}\right)^{1/2}(b_q + b_q^+). \tag{1}$$

As demonstrated by them, the deformation introduces, in addition to the parameters already present in the original problem, a new q-parameter which opens a new possibility to deform a new physical theory of interacting atoms and fields. Along this direction, instead of considering of the intensity-dependent coupling JCM, we begin with the basic interaction of field and atoms to discuss that the physical phenomenon in the system consists of interacting atoms and fields. We explicitly calculate the time evolution operator U of an atom in the deformed electromagnetic field which is dependent on deformation parameter and possesses quantum group symmetry. The density operator ρ and the time variation of photon distribution $P(n)$ are also given, both with and without intensity-dependent coupling.

We begin by writing down the Hamiltonian of the interacting system

$$H = \hbar\omega\sigma^z + \hbar\Omega b_q^+ b_q + g(\sigma^+ b_q + \sigma^- b_q^+) \tag{2}$$

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where b_q^+, b_q are the creation and annihilation operators of the q-deformed oscillator [7, 8], which possesses quantum Heisenberg group symmetry [9-11]. We separate the Hamiltonian into two commutative parts

$$H = H_1 + H_2 \tag{3}$$

where

$$\begin{aligned} H_1 &= \hbar\Omega b_q^+ b_q + \hbar\Omega\sigma^z \\ H_2 &= \hbar\Delta\Omega\sigma^z + g(\sigma^+ b_q + \sigma^- b_q^+) \end{aligned} \tag{4}$$

with $\Delta\Omega = \omega - \Omega$. And hence the development operator factorizes as

$$\begin{aligned} U(t,0) &= \exp(-\mathrm{i}Ht/\hbar) \\ &= \exp(-\mathrm{i}H_1 t/\hbar)\exp(-\mathrm{i}H_2 t/\hbar). \end{aligned} \tag{5}$$

The first part is easily diagonalized

$$\begin{aligned} U_1 &= \exp(-\mathrm{i}H_1 t/\hbar) \\ &= \exp(-\mathrm{i}\Omega b_q^+ b_q t)\begin{pmatrix} \exp(-\mathrm{i}\Omega t/2) & 0 \\ 0 & \exp(-\mathrm{i}\Omega t/2) \end{pmatrix}. \end{aligned} \tag{6}$$

The second one equals

$$\begin{aligned} U_2 &= \exp(-\mathrm{i}H_2 t/\hbar) \\ &= \sum_{n=0}^{\infty} \frac{(-\mathrm{i}t/\hbar)^n}{n!} H_2^n \\ &= \sum_{n=0}^{\infty} (-\mathrm{i})^n \frac{t^n}{n!} \begin{pmatrix} \frac{1}{2}\Delta\Omega & \Lambda b_q \\ \Lambda b_q^+ & -\frac{1}{2}\Delta\Omega \end{pmatrix}^n \end{aligned} \tag{7}$$

with $\Lambda = g/\hbar$. It is easy to show that

$$\begin{pmatrix} \frac{1}{2}\Delta\Omega & \Lambda b_q \\ \Lambda b_q^+ & -\frac{1}{2}\Lambda\Omega \end{pmatrix}^{2l} = \begin{pmatrix} (\Lambda^2 b_q b_q^+ + (\frac{1}{2}\Delta\Omega)^2)^l & 0 \\ 0 & (\Lambda^2 b_q^+ b_q + (\frac{1}{2}\Delta\Omega)^2)^l \end{pmatrix}. \tag{8}$$

It then follows that

$$\begin{pmatrix} \frac{1}{2}\Lambda\Omega & \Lambda b_q \\ \Lambda b_q^+ & -\frac{1}{2}\Lambda\Omega \end{pmatrix}^{2l+1} = \begin{pmatrix} \frac{1}{2}\Delta\Omega(\Lambda^2 b_q b_q^+ + (\frac{1}{2}\Delta\Omega)^2)^l & \Lambda(\Lambda^2 b_q^+ b_q + (\frac{1}{2}\Delta\Omega)^2)^l b_q \\ \Lambda b_q^+(\Lambda^2 b_q b_q^+ + (\frac{1}{2}\Delta\Omega)^{-2})^l & -\frac{1}{2}\Delta\Omega(\Lambda^2 b_q^+ b_q + (\frac{1}{2}\Delta\Omega)^2)^l \end{pmatrix} \tag{9}$$

which gives

$$\begin{aligned} U_2 = &\begin{pmatrix} \cos(t\sqrt{\Lambda^2 b_q b_q^+ + (\frac{1}{2}\Delta\Omega)^2}) & -\mathrm{i}\Lambda \dfrac{\sin(t\sqrt{\Lambda^2 b_q b_q^+ + (\frac{1}{2}\Delta\Omega)^2})}{\sqrt{\Lambda^2 b_q b_q^+ + (\frac{1}{2}\Delta\Omega)^2}} b_q \\ -\mathrm{i}\Lambda b_q^+ \dfrac{\sin(t\sqrt{\Lambda^2 b_q b_q^+ + (\frac{1}{2}\Delta\Omega)^2})}{\sqrt{\Lambda^2 b_q b_q^+ + (\frac{1}{2}\Delta\Omega)^2}} & \cos(t\sqrt{\Lambda^2 b_q^+ b_q + (\frac{1}{2}\Delta\Omega)^2}) \end{pmatrix} \\ &+ \begin{pmatrix} -\mathrm{i}\frac{1}{2}\Delta\Omega \dfrac{\sin(t\sqrt{\Lambda^2 b_q b_q^+ + (\frac{1}{2}\Delta\Omega)^2})}{\sqrt{\Lambda^2 b_q b_q^+ + (\frac{1}{2}\Delta\Omega)^2}} & 0 \\ 0 & +\mathrm{i}\frac{1}{2}\Delta\Omega \dfrac{\sin(t\sqrt{\Lambda^2 b_q^+ b_q + (\frac{1}{2}\Delta\Omega)^2})}{\sqrt{\Lambda^2 b_q^+ b_q + (\frac{1}{2}\Delta\Omega)^2}} \end{pmatrix}. \end{aligned} \tag{10}$$

The operators in U obey the following rules

$$\begin{aligned} \frac{\sin(t\sqrt{\Lambda^2 b_q b_q^+ + (\frac{1}{2}\Delta\Omega)^2})}{\sqrt{\Lambda^2 b_q b_q^+ + (\frac{1}{2}\Delta\Omega)^2}} b_q &= b_q \frac{\sin(t\sqrt{\Lambda^2 b_q^+ b_q + (\frac{1}{2}\Delta\Omega)^2})}{\sqrt{\Lambda^2 b_q^+ b_q + (\frac{1}{2}\Delta\Omega)^2}} \\ \cos(t\sqrt{\Lambda^2 b_q b_q^+ + (\frac{1}{2}\Delta\Omega)^2})\, b_q &= b_q \cos(\sqrt{\Lambda^2 b_q^+ b_q + (\frac{1}{2}\Delta\Omega)^2}) \end{aligned} \tag{11}$$

and their Hermitian conjugates. Using these relations it is a straightforward exercise to show that U is unitary

$$U(t,0)U^{+}(t,0)=U(t,0)U(0,t)=1 \tag{12}$$

as it should be. The density operator $\rho(t)$ can be obtained from the initial density operator by using the time development operator U

$$\rho(t)=U(t,0)\rho(0)U^{+}(t,0). \tag{13}$$

We are going to consider the situation where an atom in the appear state is injected into the cavity with one nearly resonant mode exited. We assume that this mode is in the q-deformed coherent state [7]

$$|z,q\rangle=(\exp_q|z|^2)^{-1/2}\sum_{n=0}^{\infty}\frac{z^n}{\sqrt{[n]!}}|n\rangle. \tag{14}$$

The atom stays in the cavity for a time t and is then removed. The density operator is then at time t

$$\rho(t)=U(t,0)|z,q\rangle\begin{pmatrix}1&0\\0&0\end{pmatrix}\langle z,q|U^{+}(t,0). \tag{15}$$

At the initial time $t=0$, we have a quasi-Poisson photon distribution

$$P_n(0)=\frac{(\exp_q(|z|^2))^{-1}|z|^{2n}}{[n]!}. \tag{16}$$

For later times we obtain the time variation of the distribution from (15) when we take the trace over the atomic variables

$$\begin{aligned}P_n(t)&=\mathrm{Tr}_{\mathrm{atom}}\langle n|\rho(t)|n\rangle\\&=\mathrm{Tr}_{\mathrm{atom}}\sum_{mm'}\exp(-\mathrm{i}\Omega([n]\pm\tfrac{1}{2})t)\langle n|U_2|m\rangle\begin{pmatrix}1&0\\0&0\end{pmatrix}\frac{z^m(z^*)^{m'}}{\sqrt{[m]![m']!}}\\&\quad\times(\exp_q(|z|^2))^{-1}\langle m'|U_2^{+}|n\rangle\exp(\mathrm{i}\Omega([n]\pm\tfrac{1}{2})t)\\&=\left(\cos^2(t\sqrt{\Lambda^2[n+1]+(\tfrac{1}{2}\Delta\Omega)^2})+(\tfrac{1}{2}\Delta\Omega)^2\frac{\sin^2(t\sqrt{\Lambda^2[n+1]+(\frac{1}{2}\Delta\Omega)^2})}{\Lambda^2[n+1]+(\frac{1}{2}\Delta\Omega)^2}\right)\\&\quad\times\frac{|z|^{2n}}{[n]!}(\exp_q(|z|^2))^{-1}\\&\quad+\left(\Lambda^2\frac{\sin^2(t\sqrt{\Lambda^2[n]+(\frac{1}{2}\Delta\Omega)^2})}{\Lambda^2[n]+(\frac{1}{2}\Delta\Omega)^2}\right)\frac{|z|^{2n-2}}{[n-1]!}(\exp_q(|z|^2))^{-1}.\end{aligned} \tag{17}$$

This time development contains periodic variations at the frequency $\sqrt{\varphi_n}\propto\sqrt{[n]}$, almost the same as in the case of the undeformed JCM.

The intensity-dependent coupling can be introduced as in the case of the original JC Hamiltonian

$$H_2'=\hbar\Delta\Omega\sigma^z+g(\sqrt{[N]}b_q^{+}\sigma^{-}+b_q\sqrt{[N]}\sigma^{+}). \tag{18}$$

Chaichain *et al* gave the q-analogue of HP realization [12] of the quantum $\mathrm{su}_q(1,1)$ algebra, identified as

$$K_{+}^{q}=\sqrt{[N]}b_q^{+}\qquad K_{-}^{q}=b_q\sqrt{[N]}\qquad K_0=N+\tfrac{1}{2}. \tag{19}$$

In this intensity-dependent coupling system, the time development operator is of the form

$$U_2 = \exp(-\mathrm{i}H_2't/\hbar)$$
$$= \begin{pmatrix} \cos(t\sqrt{\Lambda^2 K_-^q K_+^q + (\frac{1}{2}\Delta\Omega)}) & -\mathrm{i}\Lambda \dfrac{\sin(t\sqrt{\Lambda^2 K_-^q K_+^q + (\frac{1}{2}\Delta\Omega)^2})}{\sqrt{\Lambda^2 K_-^q K_+^q + (\frac{1}{2}\Delta\Omega)^2}} K_-^q \\ -\mathrm{i}\Lambda K_+^q \dfrac{\sin(t\sqrt{\Lambda^2 K_-^q K_+^q + (\frac{1}{2}\Delta\Omega)^2})}{\sqrt{\Lambda^2 K_-^q K_+^q (\frac{1}{2}\Delta\Omega)^2}} & \cos(t\sqrt{\Lambda^2 K_+^q K_+^q + (\frac{1}{2}\Delta\Omega)^2}) \end{pmatrix}$$
$$+ \begin{pmatrix} -\mathrm{i}\frac{1}{2}\Delta\Omega \dfrac{\sin(t\sqrt{\Lambda^2 K_-^q K_+^q + (\frac{1}{2}\Delta\Omega)^2})}{\sqrt{\Lambda^2 K_-^q K_+^q + (\frac{1}{2}\Delta\Omega)^2}} & 0 \\ 0 & \mathrm{i}\frac{1}{2}\Delta\Omega \dfrac{\sin(t\sqrt{\Lambda^2 K_+^q K_-^q + (\frac{1}{2}\Delta\Omega)^2})}{\sqrt{\Lambda^2 K_+^q K_-^q + (\frac{1}{2}\Delta\Omega)^2}} \end{pmatrix} \tag{20}$$

and the time variation of the distribution form of the photons is

$$P_n(t) = \left(\cos^2(t\sqrt{\Lambda^2[n+1]^2 + (\tfrac{1}{2}\Delta\Omega)^2}) + (\tfrac{1}{2}\Delta\Omega)^2 \frac{\sin^2(t\sqrt{\Lambda^2[n+1]^2 + (\frac{1}{2}\Delta\Omega)^2})}{\Lambda^2[n+1]^2 + (\frac{1}{2}\Delta\Omega)^2} \right)$$
$$\times \frac{|z|^{2n}}{[n]!} (\exp(|z|^2))^{-1}$$
$$+ \Lambda^2 \frac{\sin^2(t\sqrt{\Lambda^2[n]^2 + (\frac{1}{2}\Delta\Omega)^2})}{\Lambda^2[n]^2 + (\frac{1}{2}\Delta\Omega)^2} \frac{|z|^{2n-2}}{[n-1]!} (\exp_q(|z|^2))^{-1}. \tag{21}$$

In the case of resonance, i.e. $\Delta\Omega = 0$,

$$U_2 = \begin{pmatrix} \cos(t\sqrt{\Lambda^2 K_-^q K_+^q}) & -\mathrm{i}\Lambda \dfrac{\sin(t\sqrt{\Lambda^2 K_-^q K_+^q})}{\sqrt{\Lambda^2 K_-^q K_+^q}} K_-^q \\ -\mathrm{i}\Lambda K_+^q \dfrac{\sin(t\sqrt{\Lambda^2 K_-^q K_+^q})}{\sqrt{\Lambda^2 K_-^q K_+^q}} & \cos(t\sqrt{\Lambda^2 K_+^q K_-^q}) \end{pmatrix}. \tag{22}$$

The same results as those of Chaichain *et al* [4] are recovered.

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